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Curved BPS domain walls and RG flow in five dimensions

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Abstract: We determine, in the context of five-dimensional $\mathcal{N} = 2$ gauged supergravity with vector and hypermultiplets, the conditions under which curved (non-Ricci flat) supersymmetric domain wall solutions may exist. These curved BPS domain wall solutions may, in general, be supported by non-constant vector and hyper scalar fields. We establish our results by a careful analysis of the BPS equations as well as of the associated integrability conditions and the equations of motion. We construct an example of a curved BPS solution in a gauged supergravity model with one hypermultiplet. We also discuss the dual description of curved BPS domain walls in terms of RG flows.

Keywords: AdS/CFT Correspondence, Supergravity Models.

Contents

1. Introduction

According to the domain wall/QFT correspondence [1], the renormalization group flow in quantum field theories may be described in terms of a domain wall solution in dual gauged supergravity theories. An example of this is the RG flow discussed by Freedman, Gubser, Pilch and Warner [2]. This is a flow in $\mathcal{N} = 4$ super-Yang-Mills theory broken to an $\mathcal{N} = 1$ theory by the addition of a mass term for one of the three adjoint chiral superfields [3]. Its dual description is in terms of a supersymmetric domain wall solution in five-dimensional $\mathcal{N} = 8$ gauged supergravity which interpolates between two AdS vacua. The $\mathcal{N} = 2$ embedding of this UV-IR solution has been given in [4]. The supersymmetric domain wall associated to the flow of [2] is an example of a flat domain wall.

In the context of five-dimensional $\mathcal{N} = 2$ gauged supergravity, the study of BPS (i.e. supersymmetric) domain walls has been mainly restricted to the case of Ricci flat solutions [4]–[14]. It is, however, natural to ask whether there also exist curved (i.e. non-Ricci flat) supersymmetric domain wall solutions. Such domain wall solutions would presumably provide a dual gravitational description of RG flows in supersymmetric field theories in a curved spacetime. Another motivation for the study of curved domain wall solutions is given by localized gravity on anti-de Sitter domain walls embedded in AdS_5 [15]–[19].

It is known [20, 21] that field theories and also string theories in spacetimes with constant AdS curvature exhibit improved infrared behaviour. In a dual domain wall description, this improved infrared behaviour should come about by turning on AdS curvature on the domain wall. In particular, since in principle one may study the $\mathcal{N} = 1$ super-Yang-Mills theory of [2] in a spacetime with constant AdS curvature, one expects that it should be possible to construct a curved (non-Ricci flat) BPS domain wall solution in $\mathcal{N} = 2$ gauged supergravity which would provide a dual description of the RG flow. Moreover, since curved domain wall solutions have been constructed [22] in four-dimensional $\mathcal{N} = 2$ gauged supergravity, it is plausible that curved domain wall solutions also exist in five-dimensional $\mathcal{N} = 2$ gauged supergravity. Thus, whereas UV divergences in four-dimensional holographic field theories are regulated by performing computations away from the boundary of AdS_5 [23], we expect that IR divergences may be regulated by turning on curvature on the dual domain wall.

A first step towards the construction of curved supersymmetric domain wall solutions in five dimensions was taken in [24]. There we considered five-dimensional $\mathcal{N} = 2$ gauged supergravity with vector and hypermultiplets, and we analyzed the possibility of constructing curved BPS domain wall solutions which are supported by non-constant scalar fields. The construction presented in [24] is perfectly general and may be applied to curved domain walls supported by scalar fields belonging to only one type (vector or hyper) or to both types of supermultiplets. We showed that the resulting BPS equations for the warp factor and for the vector scalars are modified by the presence of a four-dimensional cosmological constant on the domain wall, extending earlier results by DeWolfe, Freedman, Gubser and Karch [25] in the context of non-supersymmetric gravitational theories in five dimensions. We also showed that the cosmological constant on the BPS domain wall must be anti-de Sitter like and that it constitutes an independent quantity, not related to any of the objects appearing in the context of very special geometry.

Related work on curved domain walls appeared in [26]. There it was argued that the integrability of the gravitini equation rules out the existence of supersymmetric domain walls with a non-vanishing cosmological constant on the wall.

Here we return to this issue and we show that curved BPS domain wall solutions may very well exist in five-dimensional $\mathcal{N} = 2$ gauged supergravity with vector and hypermultiplets. We do this by first analyzing the BPS flow equation for the hyper scalars, which we didn't give in [24]. We then use this information to check whether the integrability conditions derived from the gravitini equation associated to curved BPS domain wall solutions are satisfied, and we establish that this is indeed the case.

We then give the energy functional for curved domain wall solutions. Interestingly we find that it isn't just given in terms of squares of BPS equations and of boundary terms, but that there are also contributions that are linear in the BPS equations as well as an additional term proportional to the warp factor. The latter, whose presence was already noted in [25], is crucial for ensuring that curved solutions to the BPS flow equations also solve the Einstein equations of motion.

We use the energy functional to compute the equations of motion for the vector and the hyper scalars. The resulting equations are complicated. It is not guaranteed that a solution to the BPS flow equations will automatically solve the equations of motion for the scalar fields, as was already pointed out in [26]. This is tied to the presence, in the energy functional, of terms that are linear in the BPS equations. Thus, in order to construct curved BPS domain wall solutions one should proceed as follows. First one solves the flow equations for the warp factor and for the scalar fields. Then one plugs the resulting expressions into the equations of motion for the scalar fields and checks whether they are satisfied. Note that already in the case of flat domain walls the equations of motion yield an additional condition [4] which must be fullfilled by a solution to the BPS flow equations.

We then discuss the RG flow interpretation of curved BPS domain wall solutions.

And finally, we give an example of a curved BPS domain wall solution. We explicitly construct such a solution in a gauged supergravity model with one hypermultiplet only, and we check that the equations of motion are satisfied.

2. Five-dimensional curved BPS domain wall solutions

The five-dimensional $\mathcal{N} = 2$ gauged supergravity theories that we consider are in the class constructed in [27], describing the general coupling of n_V vector multiplets and of n_H hypermultiplets to supergravity. The scalar fields ϕ^x ($x = 1, \ldots, n_V$) of the vector multiplets parametrize a very special manifold. The hypermultiplet scalars q^X, on the other hand, parametrize a quaternionic Kähler geometry determined by $4n_H$-beins $f_X^{iA}(q^X)$, with the SU(2) index $i = 1, 2$ and the Sp($2n_H$) index $A = 1, \ldots, 2n_H$, raised and lowered by the symplectic metrics ε_{ij} and C_{AB}. We refer to [27] for more details.

The scalar potential in such theories is given by [4]

$$\mathcal{V} = -6\,W^2 + \frac{9}{2}\,g^{\Lambda\Sigma}\partial_\Lambda W\,\partial_\Sigma W \; + \frac{9}{2}\,W^2(\partial_x Q^s)(\partial^x Q^s)\,. \tag{2.1}$$

Here $g_{\Lambda\Sigma}$ denotes the metric of the complete scalar manifold, which is positive definite, involving the scalars of both the vector and the hypermultiplets. $W(\phi, q)$ and $Q^s(\phi, q)$ denote the norm and the SU(2) phases, respectively, which appear in the decomposition of the triplet of Killing prepotentials P^s, i.e. $P^s = \sqrt{3/2}WQ^s$ with $Q^sQ^s = 1$. Note that in (2.1), the derivatives acting on the SU(2) phases Q^s are only computed with respect to the scalars of the vector multiplets.

It will be convenient to rewrite (2.1) as follows,

$$\mathcal{V} = -6\,W^2 + \frac{9}{2}\,\Gamma^{-2}\,g^{xy}\partial_x W\,\partial_y W \; + \frac{9}{2}\,g^{XY}\partial_X W\,\partial_Y W\,, \tag{2.2}$$

where

$$\Gamma^{-2}(\phi, q) = 1 + W^2\,\frac{g^{xy}(\partial_x Q^s)(\partial_y Q^s)}{g^{xy}\partial_x W\,\partial_y W}\,. \tag{2.3}$$

We are interested in the construction of curved BPS domain wall solutions. These are solutions which are uncharged, which are supported by non-constant scalar fields and which have residual supersymmetry. We take the associated spacetime metric to be given by

$$ds^2 = \mathrm{e}^{2\,\mathrm{U}(r)}\hat{g}_{mn}dx^m dx^n + dr^2\,, \tag{2.4}$$

where $x^\mu = (x^m, r)$ (with $x^m = (t, x, y, z)$) and $\hat{g}_{mn} = \hat{g}_{mn}(x^m)$. We denote the corresponding tangent space indices by $a = (0, 1, 2, 3, 5)$. The metric $\hat{g}_{mn}$ is taken to be a four-dimensional constant curvature metric, i.e. $\hat{R}_{mn} = -12l^{-2}\,\hat{g}_{mn}$, with the four-dimensional cosmological constant proportional to l^{-2}. For the solution to be supersymmetric, we must take l to be imaginary [24], which corresponds to a four-dimensional anti-de Sitter spacetime. Since these solutions are uncharged, we set the gauge fields to zero.[1] We allow for a non-trivial dependence of the scalar fields on the coordinate r, and we write $\phi' = d\phi/dr$, $q' = dq/dr$ as well as $U' = dU/dr$.

A solution with residual supersymmetry is obtained by requiring that the supersymmetry transformation laws, when evaluated on a given solution, vanish for some combination of the supersymmetry transformation parameters ϵ_i. For solutions with spacetime line

[1] It can be checked that the equations of motion for the gauge fields are satisfied by these solutions.

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element (2.4), the appropriate projector condition on the supersymmetry transformation parameters ϵ_i leading to residual supersymmetry is given by [24]

$$i\gamma_5\epsilon_i = A(r)\, Q_i{}^j\epsilon_j + B(r)\, M_i{}^j\epsilon_j\,. \tag{2.5}$$

Here $Q = iQ^s\sigma^s$ and $M = iM^s\sigma^s$ denote SU(2)-valued matrices satisfying $Q_i{}^jQ_j{}^k = -\delta_i{}^k, M_i{}^jM_j{}^k = -\delta_i{}^k$ (i.e. $Q^sQ^s = 1, M^sM^s = 1$). Without loss of generality, we take Q and M to be orthogonal in SU(2) space, so that $Q^sM^s = 0$. The consistency of (2.5) then yields that

$$A^2(r) + B^2(r) = 1\,. \tag{2.6}$$

A curved BPS domain wall solution is supported by a non-trivial warp factor U as well as by non-constant scalar fields. For the solution to be supersymmetric, these various fields have to satisfy a set of so-called BPS flow equations. The flow equation for the warp factor U can be derived as follows [24]. Demanding the vanishing of the gravitini variation $\delta\psi_{mi}$ and inserting (2.5) into it yields

$$\hat{\mathcal{D}}_m\epsilon_i = \frac{i}{2}\left(U'\,A + gW\right)\gamma_m Q_i{}^j\epsilon_j + \frac{i}{2}U'B\gamma_m M_i{}^j\epsilon_j\,, \tag{2.7}$$

whose integrability gives

$$2U'\left(U' + AgW\right) = (U')^2 - 4l^{-2}\mathrm{e}^{-2U} - g^2W^2\,. \tag{2.8}$$

On the other hand, the integrability of $\delta\psi_{\mu i}$ also gives rise to

$$(U')^2 - 4l^{-2}\,\mathrm{e}^{-2U} - g^2W^2 = 0\,, \tag{2.9}$$

which yields

$$U' = \pm\gamma(r)\, gW\,, \tag{2.10}$$

where

$$\gamma(r) \equiv \sqrt{1 - \frac{4\mathrm{e}^{-2U}}{|l|^2g^2W^2}}\,. \tag{2.11}$$

Combining (2.8) and (2.9), however, also gives

$$U' = -AgW\,. \tag{2.12}$$

Then, by comparing (2.10) with (2.12) we obtain A as a function of W and of U, namely

$$A = \mp\gamma(r)\,. \tag{2.13}$$

The flow equations for the scalar fields belonging to the vector multiplets is derived as follows [24]. Demanding the vanishing of the supersymmetry variation of the gaugini subject to (2.5) yields

$$A(r)\phi^{x\,\prime} = 3g\, g^{xy}\partial_yW \tag{2.14}$$

as well as

$$B(r)M^s\phi^{x\prime} = 3\,gW\; g^{xy}\partial_yQ^s\,. \tag{2.15}$$

Combining (2.14) and (2.15) we obtain

$$A^{-2} = 1 + W^2 \frac{g^{xy}(\partial_x Q^s)(\partial_y Q^s)}{g^{xy}\partial_x W \partial_y W}\,. \tag{2.16}$$

Inspection of (2.3) then gives another expression for A in terms of the scalar fields, namely

$$A = \mp\Gamma\,. \tag{2.17}$$

The flow equation for the vector scalars thus reads

$$\phi^{x\,\prime} = \mp 3g\,\Gamma^{-1}\,g^{xy}\partial_y W\,. \tag{2.18}$$

Combining (2.15) and (2.18) yields

$$\partial_x Q^s = \mp B\Gamma^{-1}W^{-1}\partial_x W\,M^s\,. \tag{2.19}$$

By squaring (2.19) we precisely obtain (2.3).

Comparing (2.13) with (2.17) yields

$$\Gamma(\phi, q) = \gamma(r)\,. \tag{2.20}$$

Note that (2.20) only applies if a given gauged supergravity model has vector multiplets, since both the definition (2.3) and the derivation of (2.17) depend on that. For models with vector multiplets, (2.20) constitutes a consistency check on the solution, since $\Gamma(\phi, q)$ only depends on the scalar fields, whereas $\gamma(r)$ is computed from both the warp factor and the scalar fields. We observe that (2.20) is solved by $\partial_x Q^s = \mp B\gamma^{-1}W^{-1}\partial_x W\,M^s$, which is nothing but (2.19). The case of flat domain walls corresponds to $\gamma = 1$, $B = 0$ and hence $\partial_x Q^s = 0$.

The flow equations (2.10) and (2.18) were also derived in [25] in the context of non-supersymmetric five-dimensional gravity theories with a single scalar field.

Let us now turn to the flow equation for the scalar fields belonging to the hypermultiplets, which we didn't analyze in [24]. The vanishing of the hyperini equations, when subjected to (2.5), yields the following flow equation for the hyper scalars,

$$\frac{1}{3} g_{XY}\, q^{Y\,\prime} = gA\partial_X W + gWBM^s D_X Q^s\,, \tag{2.21}$$

which may also be rewritten as [26]

$$g_{XY}\, q^{Y\prime} = 3g\Sigma_X{}^Y\partial_Y W\,, \qquad \Sigma_X{}^Y = A\delta_X{}^Y + 2B\varepsilon^{rst}M^r Q^s R^t_X{}^Y\,, \tag{2.22}$$

where $R^t_X{}^Y$ denotes the SU(2) curvatures. Moreover, it can be checked that

$$\Sigma_X{}^Y g^{XZ}\Sigma_Z{}^V = g^{YV}\,. \tag{2.23}$$

Then, using (2.22) and (2.23), it follows that

$$g_{XY}\, q^{X\prime}q^{Y\prime} = 9\,g^2 g^{XY}\,\partial_X W\partial_Y W\,. \tag{2.24}$$

On the other hand, contracting (2.22) with $\partial^X W$ yields

$$q^{X\prime}\partial_X W = 3gAg^{XY}\partial_X W\partial_Y W\,. \tag{2.25}$$

Comparing (2.24) and (2.25) then gives

$$g_{XY}\,q^{X\prime}q^{Y\prime} = 3gA^{-1}q^{X\prime}\,\partial_X W = \mp 3g\gamma^{-1}q^{X\prime}\,\partial_X W\,, \tag{2.26}$$

where we used (2.13). Thus, we observe that the contracted version (2.26) of the flow equation for the hyper scalars has a similar structure as the contracted version of the flow equation for the vector scalars (2.18) given by $g_{xy}\phi^{x\prime}\phi^{y\prime} = \mp 3g\,\gamma^{-1}\,\phi^{x\prime}\partial_x W$.

Another useful relation can be obtained as follows. Contracting (2.21) with $q^{X\prime}$ and using (2.26) yields

$$q^{X\prime}BM^s\,D_X Q^s = \mp\frac{(1-\gamma^2)}{\gamma W}\,q^{X\prime}\,\partial_X W\,. \tag{2.27}$$

Now, by using $D_r Q^s = \phi^{x\prime}\,\partial_x Q^s + q^{X\prime}D_X Q^s$ and $W' = \phi^{x\prime}\,\partial_x W + q^{X\prime}\partial_X W$ as well as (2.19), (2.20) and (2.27), we obtain

$$BM^s\,D_r Q^s = \mp\frac{(1-\gamma^2)}{\gamma W}\,W'\,. \tag{2.28}$$

This concludes our discussion of the derivation of the BPS flow equations for the various fields supporting a curved BPS domain wall.

We proceed to check the various integrability conditions associated to $\delta\psi_{\mu i} = 0$. One such condition is given by (2.9). Another is given by [24]

$$3U'' + 12l^{-2}\,\mathrm{e}^{-2U} = -g_{xy}\,\phi^{x\prime}\phi^{y\prime} - \frac{1}{2}g_{XY}\,q^{X\prime}q^{Y\prime} - \frac{9}{2}\,g^2 g^{XY}\,\partial_X W\partial_Y W\,. \tag{2.29}$$

By inserting (2.10), (2.18), (2.24) and (2.26) into (2.29) we find that (2.29) is identically satisfied.

As pointed out in [26], there is one more integrability condition that needs to be checked, namely $[D_r, \hat{\mathcal{D}}_m]\epsilon_i = 0$. To evaluate this, we use the gravitini variation equation $\delta\psi_{ri} = 0$ which results in

$$D_r\epsilon_i = \frac{i}{2}gWQ_i{}^j\gamma_5\epsilon_j\,, \tag{2.30}$$

where $D_r\epsilon_i = \partial_r\epsilon_i - q^{X'}\,\omega_{Xi}{}^j\epsilon_j$. Inserting the projector condition (2.5) into the rhs of (2.30) then yields

$$D_r\epsilon_i = -\frac{1}{2}gWA\epsilon_i + \frac{1}{2}gWB(QM)_i{}^j\epsilon_j\,. \tag{2.31}$$

Using (2.7) as well as (2.31) we now evaluate $[D_r, \hat{\mathcal{D}}_m]\epsilon_i = 0$ and obtain[2]

$$\pm\gamma' + BM^s\,D_r Q^s = B^2 gW \tag{2.32}$$

as well as

$$\pm B' + \gamma Q^s\,D_r M^s = -B\gamma gW\,. \tag{2.33}$$

It is easy to check that (2.32) implies (2.33) and vice-versa. Now, using (2.11) we compute $\gamma' = B^2[\pm gW + \frac{W'}{\gamma W}]$. By inserting this as well as (2.28) into (2.32), we find that (2.32) is

[2]Our expressions differ from the ones given in [26].

identically satisfied. Thus, similarly to the case of flat domain walls, there are no additional requirements on a curved domain wall solution steming from the integrability conditions associated with $\delta\psi_{\mu i} = 0$.

Next we derive the energy functional for curved domain wall solutions (2.4). We will subsequently use it to compute the equations of motion for the various fields supporting curved BPS domain walls.

The energy functional E for non-trivial solutions of the form (2.4) may be written as follows. Denoting $V_4 = \int \sqrt{|\det \hat{g}_{mn}|}$, we can rewrite the bulk action S_{bulk} as follows,

$$\begin{aligned}
\frac{E}{V_4} \propto \frac{S_{\text{bulk}}}{V_4} = \int dr\, \mathrm{e}^{4U} \Bigg[& \frac{1}{2}(\phi^{x\prime} \pm 3\Gamma^{-1} g \partial^x W)^2 + \frac{1}{2}(q^{X\prime} - 3g\Sigma^{XY}\partial_Y W)^2 - \\
& - 6(U' \mp \gamma g W)^2 \Bigg] + \\
& + 3g \int dr\, \mathrm{e}^{4U} \left[\mp \phi^{x\prime}\partial_x W(\Gamma^{-1} - \gamma^{-1}) + q^{X\prime}(\Sigma_X{}^Y \partial_Y W \pm \gamma^{-1}\partial_X W) \right] \pm \\
& \pm \frac{12}{|l|^2} \int dr\, \mathrm{e}^{2U} \left[\frac{U'}{\gamma g W} - 1 \right] + \int dr \frac{d}{dr}\left[4\mathrm{e}^{4U} U' \mp 3\mathrm{e}^{4U}\gamma g W \right] \pm \\
& \pm \frac{12}{|l|^2} \int dr\, \mathrm{e}^{2U} \,.
\end{aligned} \tag{2.34}$$

Thus we see that the energy functional for curved walls is not just given in terms of squares of BPS equations and of total derivatives (as it is the case for flat domain wall solutions [28]), but that there are also contributions that are linear in the BPS equations as well as an additional term (the last term) proportional to the warp factor. The latter, as noted in [25], cannot be rewritten into BPS equations and total derivative terms. Its presence is, however, crucial for ensuring that curved solutions to the BPS flow equations also solve the Einstein equations of motion given by

$$\begin{aligned}
4U'' + 4(U')^2 &= -g_{\Lambda\Sigma}\phi'^{\Lambda}\phi'^{\Sigma} - \frac{2}{3}g^2\mathcal{V} \,, \\
U'' + 4(U')^2 + 12|l|^{-2}\mathrm{e}^{-2U} &= -\frac{2}{3}g^2\mathcal{V} \,.
\end{aligned} \tag{2.35}$$

Indeed, varying (2.34) with respect to U yields that $\delta_U E = 0$ when evaluated on a curved BPS solution satisfying (2.10), (2.18), (2.20), (2.22) and (2.26).

Now consider varying (2.34) with respect to a vector scalar field ϕ^x. Demanding that $\delta_x E = 0$ on a curved BPS solution, we obtain

$$\pm\left(\phi^{y\prime}\partial_y W\, \partial_x \Gamma - \gamma' \partial_x W\right) + \gamma^2 q^{X\prime}\left(\Sigma_X{}^Y \partial_x\partial_Y W \pm \gamma^{-1}\partial_x\partial_X W\right) = 0 \,, \tag{2.36}$$

where we used $\Gamma' = \gamma'$ as well as $q^{X\prime}(\partial_x \Sigma_X{}^Y)\partial_Y W = 0$ by virtue of (2.22). On the other hand, varying (2.34) with respect to a hyper scalar q^Z and demanding that $\delta_Z E = 0$ on a curved BPS solution yields

$$\begin{aligned}
&\pm \gamma^{-2}\phi^{x\prime}\partial_x W\, \partial_Z \Gamma \mp 4g\gamma W \Sigma_Z{}^Y \partial_Y W - 4gW\partial_Z W + q^{X\prime}(\partial_Z \Sigma_X{}^Y)\partial_Y W - \Sigma'_Z{}^Y \partial_Y W - \\
&\quad - q^{X\prime}\left(\Sigma_Z{}^Y \partial_X\partial_Y W - \Sigma_X{}^Y \partial_Z \partial_Y W\right) - \phi^{x\prime}\left(\pm\gamma^{-1}\partial_Z\partial_x W + \Sigma_Z{}^Y \partial_Y \partial_x W\right) = 0 \,.
\end{aligned} \tag{2.37}$$

We note that if we contract (2.36) with $\phi^{x\prime}$ and (2.37) with $\gamma^2 q^{Z\prime}$, and if we use that $\Sigma_X{}^Y = \Sigma_X{}^Y(U, \phi, q)$, then the sum of the resulting equations vanishes identically.

It can be checked that (2.36) and (2.37) are indeed nothing but the equations of motion for the scalar fields

$$\mathrm{e}^{-4U}\left(\mathrm{e}^{4U}g_{\Lambda\Sigma}\phi'^{\Sigma}\right)' - \frac{1}{2}(\partial_\Lambda g_{\Gamma\Sigma})\,\phi'^{\Gamma}\phi'^{\Sigma} = g^2\partial_\Lambda\mathcal{V} \tag{2.38}$$

when evaluated on a curved BPS domain wall solution.

If a given gauged supergravity model contains only hypermultiplets, then (2.36) is trivially satisfied, whereas (2.37) constitutes a consistency check for a given solution to the BPS flow equations (2.10) and (2.22). If, on the other hand, a gauged supergravity model contains vector multiplets (with or without hypermultiplets), then (2.36) and (2.37) may be viewed as equations which determine the second derivatives $\partial_x\partial_y Q^s$ and $\partial_x\partial_X Q^s$ through $\partial_x\Gamma$ and $\partial_X\Gamma$. These expressions will then have to agree with the ones obtained by inserting a solution to the BPS flow equations into $\partial_x\Gamma$ and $\partial_X\Gamma$. Otherwise a solution to the BPS flow equations will not solve the equations of motion for the scalar fields. In the case of flat domain walls it follows from (2.36) and (2.37) that $\partial_x\Gamma = \partial_X\Gamma = 0$, which is in accordance with $\partial_x Q^s = 0$ as derived from (2.19).

3. Curved BPS domain walls and RG flow

Let us briefly summarise the properties of curved BPS domain wall solutions that we have constructed. The BPS flow equations for the warp factor and for the scalar fields are given by

$$\begin{aligned}
U' &= \pm\gamma g W\,,\\
\phi^{x\,\prime} &= \mp 3g\,\gamma^{-1}\,g^{xy}\partial_y W\,,\\
q^{X\prime} &= 3g\,g^{XY}\Sigma_Y{}^Z\partial_Z W\,,
\end{aligned} \tag{3.1}$$

where $\gamma(r) = \sqrt{1-4|l|^{-2}(\mathrm{e}^U gW)^{-2}}$ and where $\Sigma_X{}^Y = A\delta_X{}^Y + 2B\varepsilon^{rst}M^r Q^s R^t_X{}^Y$. Here $A = \mp\gamma$, $A^2 + B^2 = 1$, and $\Sigma_X{}^Y$ satisfies $\Sigma_X{}^Y g^{XZ}\Sigma_Z{}^V = g^{YV}$. In the case that a given gauged supergravity model contains vector multiplets, a solution to the BPS flow equations (3.1) will also have to satisfy the following consistency condition due to supersymmetry,

$$\Gamma(\phi, q) = \gamma(r)\,, \tag{3.2}$$

or equivalently,

$$\partial_x Q^s = \mp B\gamma^{-1}W^{-1}\partial_x W\, M^s\,. \tag{3.3}$$

If a gauged supergravity model contains only hypermultiplets, then this consistency condition is absent.

It is not guaranteed that a solution to the BPS flow equations (3.1) will also automatically solve the scalar equations of motion (2.36) and (2.37). Thus, in order to construct a curved BPS domain wall solution one should proceed as follows. First one constructs a solution to the BPS flow equations (3.1). In the presence of hypermultiplets, this requires choosing a triplet M^s such that $M^sQ^s = 0, M^sM^s = 1$. It may be that in order to explicitly construct a solution to (3.1), one has to expand γ and $\Sigma_X{}^Y$ in powers of $|l|^{-1}$ and solve the flow equations iteratively as a power series in $|l|^{-1}$. Then one checks whether the consistency condition (3.3) is met by this solution (this only applies to models which include vector multiplets). If so, then one finally checks the scalar equations of motion (2.36) and (2.37). If they are satisfied, one has managed to construct a curved BPS domain wall solution.

Observe that, on a solution to the BPS flow equations (3.1), the potential (2.2) may be written in a more symmetric way, namely as

$$\begin{aligned}\mathcal{V} &= -6\,W^2 + \frac{9}{2}\gamma^{-2}\,g^{xy}\partial_x W\,\partial_y W \;+ \frac{9}{2}\,\Sigma_Z{}^X g^{ZV}\Sigma_V{}^Y\partial_X W\,\partial_Y W \\ &= -6\,W^2 + \frac{1}{2}\,g_{xy}\,\phi^{x\prime}\phi^{y\prime} + \frac{1}{2}\,g_{XY}\,q^{X\prime}q^{Y\prime}\,. \end{aligned} \tag{3.4}$$

Let us now turn to the RG flow interpretation of a curved BPS domain wall solution. In the case of flat domain walls with line element $ds^2 = \mathrm{e}^{2U}\eta_{mn}dx^m dx^n + dr^2$, the renormalisation group scale μ of the dual field theory is usually identified with $\mu = \mathrm{e}^U$ [2], where $0 < \mu < \infty$. The field theory UV region ($U \to \infty, \mu \to \infty$) is identified with $r \to \infty$. This identification amounts to choosing the upper sign in the flow equations (3.1). A flow towards the infrared in field theory ($U \to -\infty\,,\ \mu \to 0$) then corresponds to a a flow towards smaller values of r.

In the case of curved domain walls, we again identify e^U with the renormalisation group scale μ of the dual field theory, $\mu = \mathrm{e}^U$. Now, however, it may happen that U cannot run anylonger over the whole range $-\infty < U < \infty$. This is tied to the fact that γ, which can have a value between $0 \leq \gamma \leq 1$, becomes vanishing whenever $\mathrm{e}^U = \frac{2}{|l|}(gW)^{-1}$. Since both W and U are functions of r, this will happen at specific values of r. In order to discuss the various possibilities, let us introduce $T = (\mathrm{e}^U gW)^{-1}$ and $\Lambda^2 = 4|l|^{-2}$, so that $\gamma(r) = \sqrt{1 - \Lambda^2 T^2(r)}$. Let us then consider deforming a flat domain wall solution (which has $\gamma = 1$) by turning on Λ^2 (which corresponds to turning on curvature on the domain wall). There are then two possibilities which we will now discuss.

The first possibility consists in the following. When turning on a small amount Λ^2, $\gamma(r)$ remains non vanishing along the flow. That is, for small values of Λ^2, $T^2(r)$ has the property that $T^2(r) < \Lambda^{-2}$ along the curved solution. This curved solution exists over the whole range of r. Then, when increasing Λ^2, there will be a critical value Λ_c^2 at which $T^2(r_c) = \Lambda_c^{-2}$, i.e. $\gamma(r_c) = 0$ at a certain position $r = r_c$. At this critical value of Λ^2, there is yet no obstruction in constructing a curved solution over the whole range of r. Now let us continue to increase the value of Λ^2 (corresponding to a large amount of curvature on the wall). Then $T^2(r) > \Lambda^{-2}$ in a certain region of r, i.e. γ becomes imaginary in that region. This region will be delimited by the zeros of γ. In this region there is no real solution to the curved BPS equations. The curved domain wall solution can only be constructed outside the region where γ is becoming imaginary. The associated warp factor will now only cover part of the range $-\infty < U < \infty$. As we continue to increase the value of Λ^2, the forbidden region becomes larger and larger, and the region where the solution exists smaller and smaller. An example of a curved domain wall solution exhibiting these features will be given in section 4.

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The second possibility is the following. In contrast to the one discussed above, now, as soon as one turns on a small amount of curvature on the wall, γ develops a zero somewhere, i.e. $\gamma(r_{\rm IR}) = 0$ at $T^2(r_{\rm IR}) = \Lambda^{-2}$. Let us assume that $T(r)$ is a monotonic function. Then the value $r_{\rm IR}$ delimits two regions. In the range $r_{\rm IR} \leq r < \infty$, γ has the property that $\gamma \geq 0$, whereas for values $r < r_{\rm IR}$ γ becomes imaginary. A real solution to the curved BPS equations then only exists in the range $r_{\rm IR} \leq r < \infty$. The warp factor U runs over the range $U_{\rm IR} \leq U < \infty$, with $U_{\rm IR}$ determined by $U_{\rm IR} = \log[\Lambda(gW(r_{\rm IR}))^{-1}]$. Thus, by identifying e^U with an RG energy scale μ, we see that a non-vanishing four-dimensional cosmological constant Λ^2 acts as an infrared cutoff in the field theory, i.e. $\mu_{\rm IR} = \mathrm{e}^{U_{\rm IR}} \leq \mu < \infty$. A curved domain wall then provides a dual gravitational description of an RG flow in a field theory on an AdS_4 space, with its curvature acting as an infrared regulator [20]. Observe that, as $\Lambda \to \infty$, $\mu_{\rm IR} \to \infty$ (provided that W remains finite). Cranking up the curvature on the domain wall thus pushes the infrared regulator towards the UV region.

Let us now proceed and show that a c-function also exists for curved BPS domain wall solutions. Using the flow equations (3.1) as well as (2.26) we compute

$$U'' - 4|l|^{-2}\mathrm{e}^{-2U} = \gamma^{-1}W' = -3g^{\Lambda\Sigma}\partial_\Lambda W \partial_\Sigma W - 3(\gamma^{-2}-1)\, g^{xy}\partial_x W \partial_y W \leq 0\,, \qquad (3.5)$$

where Λ, Σ run both over the vector and the hyper scalars. Hence we conclude that W is non decreasing along the flow towards smaller values of r. We also note that the lhs of (3.5) is nothing but $\frac{1}{3}(R_r{}^r - R_t{}^t)$, which equals $\frac{1}{3}(T_r{}^r - T_t{}^t)$ through the Einstein equations [2]. Equation (3.5) then translates into $T_r{}^r - T_t{}^t \leq 0$, which yields the weaker positive energy condition [2]. Thus, in the presence of a four-dimensional cosmological constant on the wall, $C(r) \propto W^{-3}$ continues to play the role of a c-function, i.e. of a function that is non increasing along the flow towards the infrared [29, 2].

Using the above gravitational description of an RG flow, the associated beta-functions are given by

$$\beta^\Lambda = \mu \frac{\partial \phi^\Lambda}{\partial \mu} = \gamma^{-1}W^{-1}\phi'^\Lambda\,, \qquad (3.6)$$

where Λ runs over both the vector and the hyper scalars. The field theory has a conformal fixed point whenever $\phi'^\Lambda = 0$. Inspection of (3.1) then shows that $\partial_x W = \partial_X W = 0$ at a fixed point. Near a fixed point $\phi^\Lambda_{\rm fix}$ we can write the BPS flow equations (3.1) for the scalar fields as $(\delta\phi^\Lambda)' = M^\Lambda{}_\Sigma\, \delta\phi^\Sigma$, where $\delta\phi^\Sigma = \phi^\Sigma - \phi^\Sigma_{\rm fix}$. Here $M^\Lambda{}_\Sigma$ denotes a constant matrix with finite eigenvalues. Hence we conclude that a fixed point $\phi^\Lambda_{\rm fix}$ may only be reached as $|r| \to \infty$. For fixed scalars, the curved BPS flow equation for U can be solved, resulting in $\mathrm{e}^U = \Lambda(gW)^{-1}\cosh(gW(r - r_0))$ [25]. As $|r| \to \infty$ we then obtain that $\mathrm{e}^U \to \infty$. Hence we conclude that for curved domain walls the fixed points can only be approached in UV-like directions ($\mu = \mathrm{e}^U \to \infty, \gamma \to 1$). This is in accordance with the expectation that turning on a cosmological constant on the wall does not affect the fixed-point behaviour of the dual field theory in the UV. On the other hand, as $\gamma \to 0$, we see from (3.6) and (3.1) that in general (some of) the beta-functions will blow up.

Finally, let us briefly comment on the energy functional (2.34). When computing it on a curved BPS domain wall solution, one finds that the boundary terms as well as the last term in (2.34) are divergent at $r = \infty$. In the case that there is a fixed point there, i.e. W = constant and $U = gWr$ at $r = \infty$, these divergences may be removed by boundary counterterms of the form $W\sqrt{|\det g_{mn}|}$ and $W^{-1}\sqrt{|\det g_{mn}|}R$, where R denotes the intrinsic curvature tensor computed from the induced metric $g_{mn} = \mathrm{e}^{2U}\hat{g}_{mn}$ [30, 31].

4. An example

Let us now explicitly construct a curved domain wall solution in a specific gauged supergravity model based on the coupling of the universal hypermultiplet to supergravity. The associated quaternionic Kähler space is given by $\frac{\mathrm{SU}(2,1)}{\mathrm{SU}(2)\times\mathrm{U}(1)}$. This space can be parametrized by the four hyper scalars $q^X = (V, \sigma, \theta, \tau)$, and in this parametrization its metric is given by $ds^2 = \frac{1}{2}V^{-2}dV^2 + \frac{1}{2}V^{-2}(d\sigma - 2\tau d\theta + 2\theta d\tau)^2 + 2V^{-1}(d\theta^2 + d\tau^2)$. We perform the gauging of the isometry associated with the U(1) transformation $C \to \mathrm{e}^{i\phi}C$, where $C = \theta - i\tau$. This U(1) is part of the isotropy group of the quaternionic manifold and therefore its gauging gives rise to a W with a critical point [4]. The associated triplet of Killing prepotentials P^s is given by [4]

$$P^s = \sqrt{6}\left(-\frac{\theta}{\sqrt{V}}, -\frac{\tau}{\sqrt{V}}, \frac{1}{2} - \frac{\theta^2 + \tau^2}{2V}\right). \tag{4.1}$$

Then $W = 1 + \frac{\theta^2+\tau^2}{V}$ and

$$Q^s = \left(-\frac{2\sqrt{V}\theta}{V + (\theta^2 + \tau^2)}, -\frac{2\sqrt{V}\tau}{V + (\theta^2 + \tau^2)}, \frac{V - (\theta^2 + \tau^2)}{V + (\theta^2 + \tau^2)}\right). \tag{4.2}$$

Observe that W has a critical point at $\theta = \tau = 0$ ($\partial_X W = 0$).

In the flat case ($\gamma = 1, B = 0$), a solution to the BPS flow equations (3.1) is given by

$$\sigma_0 = \theta_0 = 0\,, \qquad \tau_0 = \sqrt{1 - V_0}\,, \qquad V_0 = 1 - \mathrm{e}^{-6r}\,, \qquad U_0 = \frac{1}{6}\log(\mathrm{e}^{6r} - 1)\,, \tag{4.3}$$

where we set $g = 1$ and where we picked the upper sign in (3.1). The above trivially satisfies the equations of motion (2.37) for the hyper scalars. We note that this flat solution is supported by one hyper scalar field, since $V_0 + \tau_0^2 = 1$. This solution possesses a fixed-point at $r \to \infty$ where $U_0 \to r$, and a curvature singularity at $r = 0$ where $U_0 \approx (6r)^{1/6}$. The range of r is thus restricted to $0 < r < \infty$.

Now consider perturbing this flat domain wall solution by turning on curvature on the domain wall. In order to solve the curved BPS flow equations (3.1), we need to specify the triplet M^s which enters the definition of $\Sigma_X{}^Y$. Let us introduce the vector

$$\vec{\omega} = \left(0, \frac{Q^3}{\sqrt{(Q^2)^2 + (Q^3)^2}}, -\frac{Q^2}{\sqrt{(Q^2)^2 + (Q^3)^2}}\right), \tag{4.4}$$

which satisfies $\vec{\omega}\cdot\vec{\omega} = 1$ and $\vec{\omega}\cdot\vec{Q} = 0$. We then take $\vec{M}$ to be given by $\vec{M} = \vec{\omega}$. For this choice of $\vec{M}$, we can consistently truncate the model by setting $\sigma = \theta = 0$. In doing so, not only do we find that the BPS equations for σ and θ are automatically solved, but also the equations of motion (2.37) for *all* the hyper scalars are identically satisfied (to all orders in $|l|^{-1}$)!

Having thus checked the equations of motion, we return to the curved BPS flow equations for the remaining fields U, V and τ, which are given by

$$
\begin{aligned}
U' &= \gamma\,(1+f)\,V^{-1}\,, \\
V' &= 6\gamma\,(1+f-V) + 12|l|^{-1}\,\mathrm{e}^{-U}\sqrt{1+f-V}\,\frac{V^{3/2}}{1+f}\,, \\
f' &= 12|l|^{-1}\,\mathrm{e}^{-U}\sqrt{1+f-V}\,\sqrt{V}\,,
\end{aligned}
\tag{4.5}
$$

where we set $\tau = \sqrt{1+f-V}$. Then, introducing the combinations

$$
\begin{aligned}
T &= \mathrm{e}^{-U} V (1+f)^{-1}\,, \\
h &= V (1+f)^{-1}\,,
\end{aligned}
\tag{4.6}
$$

we obtain that $\gamma = \sqrt{1-4|l|^{-2}T^2}$ as well as $T' = \gamma\, T(5h^{-1}-6)$ and $h' = 6\gamma\,(1-h)$. From these equations we infer that

$$
\frac{T'}{T} = \frac{h'\,(5-6h)}{6h(1-h)}\,. \tag{4.7}
$$

Integrating (4.7) yields

$$
T = h^{5/6}(1-h)^{1/6}\,, \tag{4.8}
$$

where we fixed the integration constant by demanding that (4.8) reduces to the appropriate expression for the flat solution (4.3). Thus we obtain that

$$
\mathrm{e}^{-U} = \frac{T}{h} = \left(h^{-1}-1\right)^{1/6}\,, \tag{4.9}
$$

where $h(r)$ satisfies the following differential equation by virtue of the various relations derived above:

$$
h' = 6(1-h)\sqrt{1-4|l|^{-2}\,h^{5/3}(1-h)^{1/3}}\,. \tag{4.10}
$$

The differential equation for f may be rewritten as follows:

$$
(\log(1+f))' = 12|l|^{-1}h^{1/3}(1-h)^{2/3}\,. \tag{4.11}
$$

Equations (4.9)–(4.11) determine the curved BPS domain wall solution to all orders in the cosmological constant. The function $h(r)$ can be obtained by numerical integration of (4.10). Inserting it into (4.9) and (4.11) then yields $\mathrm{U}(r), f(r)$ as well as $V(r) = h(r)(1+f(r))$ and $\tau(r) = \sqrt{(1-h(r))(1+f(r))}$. Thus, in contrast to the flat solution (4.3), the curved solution is supported by two hyper scalar fields.

Inspection of the relation $T^6 = h^5(1-h)$ shows that $h(r)$ has the range $0 \le h \le 1$. From $h' = 6\gamma\,(1-h)$ we then infer that $h(r)$ is a monotonic function in r, and hence also $\mathrm{e}^{\mathrm{U}(r)}$. The curved solution possesses the same fixed point and singularity structure as the flat solution (4.3). Namely, the point $h = 1$ corresponds to $f = \text{const}$, $V = \text{const}$, $\tau = 0$, $\mathrm{e}^U = \infty$, and hence to a fixed point at $r = \infty$. The point $h = 0$, on the other hand, corresponds to a curvature singularity at $r = 0$, where $h \approx 6r$ and $U \approx (6r)^{1/6}$. Observe that $\gamma = 1$ at both $h = 0$ and $h = 1$.

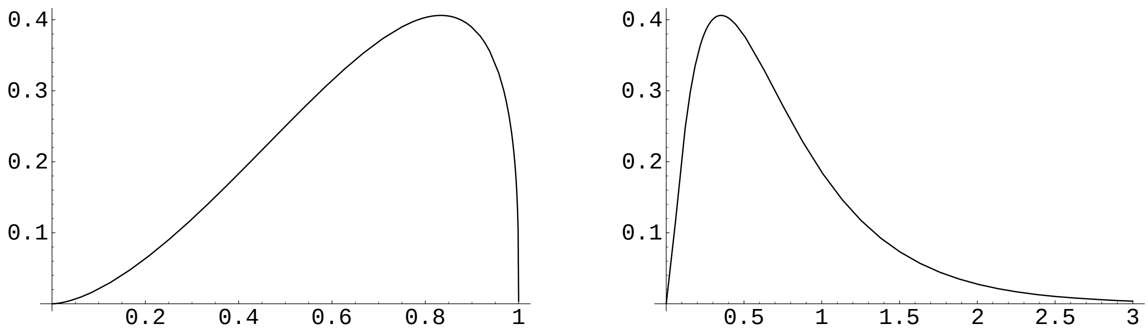

Figure 1: Left: $T^2(h)$. Right: $T^2(r)$ for the value $\Lambda = 1$.

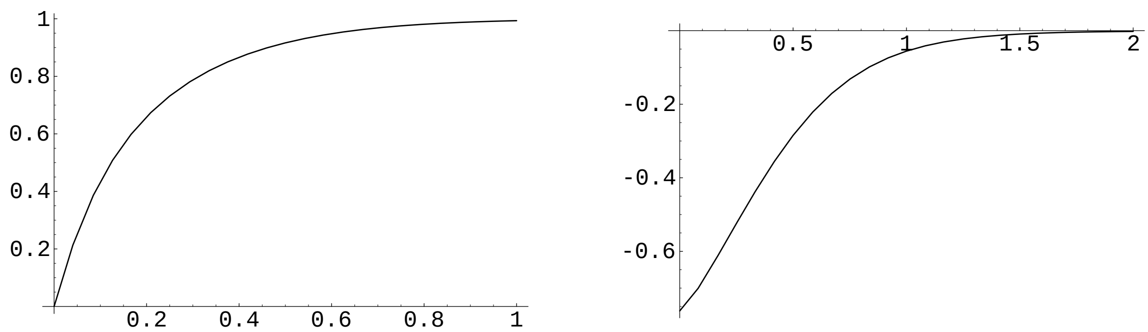

Figure 2: $h(r)$ (left) and $f(r)$ (right) for the value $\Lambda = 1$.

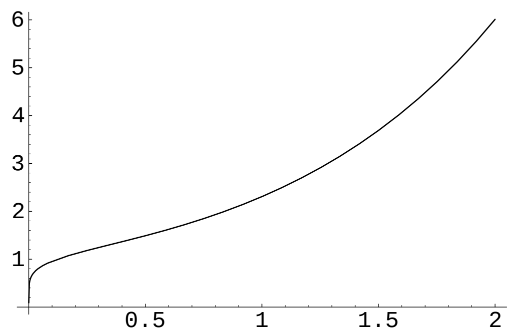

Figure 3: $\mathrm{e}^{\mathrm{U}(r)}$ for the value $\Lambda = 1$.

Using (4.8), we can plot T^2 over h. We see that T^2 has a maximum whose value doesn't exceed 0.41 (see figure 1). Hence it follows that for a small value of $\Lambda^2 = 4|l|^{-2}$ the function $\gamma = \sqrt{1 - \Lambda^2 T^2}$ is positive, $\gamma > 0$, and the curved BPS solution exists for the entire range $0 < r < \infty$. It can be obtained by numerical integration of (4.10) and (4.11). In figures 1, 2 and 3 we have plotted $T^2(r), h(r), f(r)$ and $\mathrm{e}^{\mathrm{U}(r)}$ for the value $\Lambda = 1$.

When increasing the value of Λ^2, there is a critical value Λ_c^2 at which Λ_c^{-2} equals the maximum of $T^2(h)$ ($T^2_{\max} = \Lambda_c^{-2}$). Thus there is a particular point $r = r_c$ where $\gamma(r_c) = 0$. Since $\gamma(r) \geq 0$ over the range $0 < r < \infty$, there is yet no obstruction to having a curved solution over the entire range of r. However, when continuing to increase the value of Λ^2

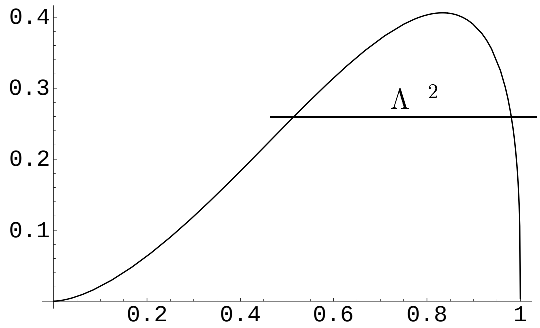

Figure 4: The two real solutions of $h^5 - h^6 = \Lambda^{-6}$ occur where the horizontal line Λ^{-2} intersects $T^2(h)$.

($\Lambda^2 > \Lambda_c^2$), the maximum of $T^2(h)$ becomes much larger than Λ^{-2}. Then γ develops two zeros determined by the two real solutions of the equation $h^5 - h^6 = \Lambda^{-6}$ (see figure 4). Let us denote these two solutions by h_I and h_{II}, where $h_I < h_{II}$. They occur at two specific values for r, r_I and r_{II}, with $r_I < r_{II}$. Thus, in the region $h_I < h < h_{II}$ (corresponding to $r_I < r < r_{II}$) γ is imaginary, and there does not exist a real curved solution to the BPS flow equations. The curved domain wall solution only exists in the regions $0 < h \leq h_I$ and $h_{II} \leq h < 1$ corresponding to $0 < r \leq r_I$ and $r_{II} \leq r < \infty$, respectively.

The curved BPS solution constructed above made use of a specific choice for $\vec{M}$, namely $\vec{M} = \vec{\omega}$. In principle there may exist other choices for $\vec{M}$ which also lead to curved BPS solutions. Let us now pick a more general $\vec{M}$, namely $\vec{M} = c(r)\vec{\omega} + \sqrt{1 - c^2(r)}\,\vec{\omega} \times \vec{Q}$. When evaluated on the flat solution (4.3), $\vec{M}$ is given by

$$\vec{M} = \left(\sqrt{1 - c^2(r)}, c(r)\,(-1 + 2V_0), 2c(r)\,\sqrt{1 - V_0}\sqrt{V_0}\right). \tag{4.12}$$

For this more general $\vec{M}$, and in contrast with the previous choice, we are only able to construct the curved domain wall solution order by order in $|l|^{-1}$. To be specific, let us then determine the lowest order modification of the flat solution. Since $B = 2|l|^{-1}\mathrm{e}^{-U}W^{-1}$, the lowest order modification is of order $|l|^{-1}$. At this order, $B = 2|l|^{-1}\mathrm{e}^{-U_0}V_0$ and $\gamma = 1$. We now determine the proportionality function $c(r)$ by demanding that the equations of motion (2.37) for the hyper scalars be satisfied to order $|l|^{-1}$. Observe that we may then use the flat domain wall solution in (2.37), since to order $|l|^{-1}$ each term comes explicitly multiplied by $|l|^{-1}$. We find that to order $|l|^{-1}$ all terms in (2.37) cancel out provided that c is constant!

Having determined M^s to order $|l|^{-1}$, we now turn to the BPS flow equations (3.1). We again set $\tau = \sqrt{1 + f - V}$. Expanding V and U as $V = V_0 + V_1$ and $U = U_0 + U_1$, we find that to order $|l|^{-1}$ the BPS flow equations are solved by

$$\begin{aligned} U_1 &= 0\,, \qquad V_1 = -c\,|l|^{-1}(1 - \mathrm{e}^{-6r})\,H(r)\,, \\ \sigma &= \sqrt{1 - c^2}\,|l|^{-1}H(r)\,, \qquad \theta = \frac{1}{2}\sqrt{1 - c^2}\,|l|^{-1}\mathrm{e}^{-3r}\,H(r)\,, \\ f &= -c\,|l|^{-1}H(r)\,, \end{aligned} \tag{4.13}$$

where

$$H(r) = 2\mathrm{e}^{-4r}(1 - \mathrm{e}^{-6r})^{1/3} + \mathrm{e}^{-4r}\,{}_2\mathrm{F}_1\left(\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \mathrm{e}^{-6r}\right), \tag{4.14}$$

and where we set various integration constants to zero. Thus, to lowest order in $|l|^{-1}$, (4.13) (together with (4.3)) describes a curved domain wall solution that solves both the BPS flow equations and the equations of motion. This curved solution is supported by at least two hyper scalar fields. In the case that $c = 1$, the curved solution (4.13) is in agreement with (4.9)–(4.11) when expanded to lowest order in $|l|^{-1}$.

As a further application of our results, it would be interesting to construct a curved version of the flow discussed in [4]. This flow, which is an $\mathcal{N} = 2$ version of the FGPW flow [2], is described by a flat wall which is supported by one vector scalar and one hyper scalar. We observe that the triplet M^s, which is necessary for the construction of the curved wall, can now be determined via (3.3). Yet another interesting application would be to construct a curved version of the GPPZ flow [6]. We hope to return to these issues in the future.

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